Errata: A simple model describing a pure QED universe

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(1) Proton parity (bottom of page 41)

Contrary to what is stated on page 41, the antiproton has negative parity.

(2) Higgs contribution to the lepton mass (pages 6, 23, 33-34, 39-44)

The result in Appendix C is wrong: the Higgs boson's contribution to the mass of the charged lepton is positive — not negative. Consequently, the conclusion in Appendix E.8 that the appearance of the heavy Higgs particle causes a decrease in lepton mass and an increase in quark energy is wrong, too. For a correct derivation of the Higgs boson's contribution to the lepton mass, see page 11 below.

In phase 3, the mass of the charged lepton (τ, μ, e) is of purely electromagnetic origin, generated entirely by massless virtual photons. When new particles (H, Z^0, ν, $W^{\pm}$) contributing to the mass of the already existing charged lepton appear, the energy of the virtual photons drops so that the lepton's mass remains unchanged as required by the law of conservation of energy. Therefore, when the Higgs first appears, the lepton mass splits into an electromagnetic mass component and a small component generated by virtual Higgs particles, which acquire masses that equal their contributions to the lepton mass [1]:

$$\begin{aligned} m_{H_\tau} &= 0.505 \text{ MeV}, \\ m_{H_\mu} &= 106.085 \text{ eV}, \\ m_{H_e} &= 12.0006\ \mu\text{eV}. \end{aligned} \tag{0.1}$$

Energy is transported from the background radiation to the quarks by the light Higgs (H_τ, H_μ, H_e). After being emitted by the leptons appearing in the propagators of the background photons, the Higgs particles are absorbed by the quarks, who use their energy to build first the pion and later the proton.

Soon after the first appearance of the light Higgs, the heavy Higgs (with mass about 125 GeV) shows up with the task of generating the neutral Z boson's mass (91.19 GeV).

In summary, the simulation of the universe's first femtoseconds suggests that the perfect symmetry of literally nothing is broken by the appearance of the massive, neutral, and spinless Dirac particle — a kind of relativistic harmonic oscillator built up in the time t_c, and attempting to annihilate itself in the same time. Consequently, the D-particle's lifetime equals the initial age of the physical universe, $t_c \approx 10^{-19}$ s.

With a massless expanding universe forbidden by the law of conservation of energy, the unstable matter content of the expanding universe undergoes repeated symmetry-breaking transformations until a stable proton–electron pair (pe^-) finally appears when the age of the universe is nearly 40 000 t_c, or approximately 4×10^{-15} seconds:

$$D \to \tau_0^+\tau_0^- \to \underline{\mu}_0^+\underline{\mu}_0^- \to \mu_0^+\mu_0^- \to \underline{e}^+\underline{e}^- \to e^+e^- \to HZ\nu W \text{ mechanism} \to p\bar{p} \to pe^-$$

1   9   0   23   0   37 293   1000   0 ∞

1   10^2 10^2 10^3 $2{\times}10^3$ $3{\times}10^9$

The underlined symbols indicate newborn, "frozen" particles, which immediately turn into dynamically interacting particles. The first row of numbers shows duration of existence of the particle pairs in units of t_c. The next row indicates the number of particles at the beginning and end of each phase. Thus, phase 1 begins with one D particle and ends with almost 100 photon pairs resulting from tauon-pair annihilation ($\tau_0^+\tau_0^- \to \gamma_\tau\gamma_\tau$). Phase 2 begins with nearly 100 spinless-muon pairs and ends with about 1000 photon pairs. Phase 3, in turn, begins with about 2000 electron pairs (rematerialized pairwise from pairs of photons: $\gamma_\mu\gamma_\mu \to e^+e^-\ e^+e^-$) and ends with approximately 3×10^9 background photons (γ).

No transfer of energy or mass takes place in the first three phases. However, the final transition from four mass-bearing electron pairs to one electron–proton pair requires repeated transport of energy from the background photons to the proton-building quarks. This is where the Higgs–neutrino (or $HZ\nu W$) mechanism comes in:

$$e^+e^-e^+e^- \to \underline{\pi}^+\underline{\pi}^-\underline{\pi}^+\underline{\pi}^- \xrightarrow{H} \pi_-^+\pi_-^-\pi_-^+\pi_-^- \to \pi_-^+\pi_-^- \xrightarrow{Z} \pi_+^+\pi_-^- \to \pi_+^+\pi_+^- \xrightarrow{\nu W} p_+\bar{p}_+ \to p_+e_+^-$$

0   10^{-5} 10^{-5} 10^3 10^{-5} 10^{-5} ∞

Again, the numbers indicate time duration, suggesting that the process takes about $10^3\, t_c$, or 10^{-16} seconds. The uppermost row shows in which reaction each weakly interacting particle first appears.

After the last two electron pairs have transformed into equally heavy "frozen" pion pairs (underlined), the mass (or rest energy — no kinetic energy exists) needed to turn the pions into dynamically interacting physical particles is brought by the Higgs from the background radiation. That is, the Higgs is forced to appear on the scene, extract mass from virtual leptons appearing in the propagators of the background photons, and hand it over to u and d quarks forming four massive real pions.

Within less than 10^{-5} time units t_c (with $t_c \approx 10^{-19}$ s) from its creation, one of the pion pairs annihilates via strong interaction.

After a lapse of another $10^{-5}\, t_c$, the imminent annihilation of the remaining pion pair is prevented by the neutral Z boson (whose mass is generated by the heavy Higgs) coming to the rescue — switching the intrinsic parity (indicated by subscript: $\pi_-^+ \to \pi_+^+$) of one of the pions, thereby making strong decay of the pair impossible. However, in about $10^3\, t_c$, the weak parity-switching force introduced by the Z causes a second change of pion intrinsic parity, which again enables strong decay of the pion pair.

The comparatively long time ($10^3\, t_c$) that elapses between the two parity-switching events introduces a particle–antiparticle asymmetry (point 4.27 on page 60) that should reveal itself in various ways as a kind of "superweak" effect. Presumably, the CP-violating "superweak force" acting in kaon and B-meson decay is a consequence of this matter–antimatter or, more precisely, d–$\bar{d}$ asymmetry.

With the last real pion pair doomed to disappear, its natural replacement is a real proton–antiproton pair. Consequently, the mass transport has to be repeated three more times.

Part of the delivered mass remains unused by the quarks and must be restored to the leptons — the natural alternative in an indeterminate quantum universe where kinetic energy does not exist. The unused mass is returned by spin-$\frac{1}{2}$ neutrinos which, to be able to travel from quarks to leptons, require help of both the Z particle and yet another new type of particle — the charged W boson, with the bulk of its mass generated by the heavy Higgs.

With the matter of the universe concentrated in a single proton–antiproton pair, the pair's annihilation is forbidden by the law of conservation of energy. Therefore, the antiproton is forced to transform into an electron — an event that finally results in a viable world containing stable proton–electron matter. The real antiproton's parity, which it inherits from the pion, has the wrong sign (since the proton and antiproton, which already exist before the real pair is created, have positive and negative parity, respectively). Therefore, the antiproton's decay does not involve a change of parity, and the fact that the decay takes place does not prove that ordinary antiprotons or protons should be able to decay, too.

The decay of the heavy antiproton into a light electron and radiation causes the proton and electron to move relative to each other. Consequently, it leads to the introduction of kinetic energy and heat, and thereby to activation of the second law of thermodynamics as well as to activation of the gravitational force, which has no role to play as long as the universe is in an indefinite quantum state, where position, motion, and kinetic energy are not defined.

See page 10 below for a summary of the information about the muon mass that is given by the simulation of the universe's early phases.

(3) Dark matter (pages 14, 19)

One big mistake is my assumption (point C.11 on page 54) that Jupiter-mass or smaller black holes gives the universe a density of

$$\rho_u = \frac{3H^2}{4\pi G}, \qquad (5.15)$$

which is twice the so-called critical density. Because recent observations [5] show that the contributions of black holes to the universe's dark matter is small, Eq. (5.15) is by necessity wrong. Consequently, Sections 5 and 7 have to be reconsidered.

In addition to Eq. (5.15), where H is the Hubble expansion rate and G the gravitational constant, equations of interest are

$$H = \frac{c}{R} = \frac{32Gm_e}{B^2 r_{cl}^2\, c} = 56.8\ \mathrm{km\,s^{-1}\,Mpc^{-1}}, \tag{7.17}$$

where R is the radius of the universe defined via $H = c/R$,

$$1/H = 17.2\ \mathrm{Gyr}, \tag{7.18}$$

$$H = 1/3t, \tag{7.22}$$

where t is the age of the universe, and

$$R = 3ct. \tag{7.23}$$

From Eqs. (7.17) and (7.22) it follows that G, H, and R change with time according to

$$\dot{G}/G = \dot{H}/H = -\dot{R}/R = -1/t = -3H. \tag{7.24}$$

Another indication that something is fundamentally wrong comes from the fact that Eq. (7.22) gives an unrealistically low value, $t = 1/3H = 5.7$ Gyr, for the age of the universe.

The conclusion that the universe is dominated by invisible dark matter results from the assumption that gravity and expansion balance each other — an assumption which leads to the prediction that the density of the universe must be very close to the so-called critical density, which is $\rho_c = 3H^2/8\pi G$.

In contrast, the present theory, which results from applying the momentum equation to space, and which states that gravity is but an inevitable consequence of the expansion, does not put any restrictions on the value of ρ_u.

So, why should the value in Eq. (5.15) be correct when the age t resulting from the same calculations proves to be much too low? The logical conclusion is that the astronomers have already identified most of the dark matter, and that in reality the density is only about five percent of the "critical mass." Consequently, the density of the universe is expected to be about 2.5 percent, or one fortieth of the value obtained in Eq. (5.15).

But if that is so, how is it possible that the same calculation gives a value for the Hubble constant H that agrees fairly well with the observed value? The answer is that it is possible because gravity is generated by the universe's expansion, which means that the rate of expansion at a given point of time determines the strength of the gravitational force at the same point in time. Therefore, if the Hubble expansion rate is known, the value of G follows immediately from Eq. (7.17). Conversely, when G is known, H is obtained from the same equation.

Unlike G, which has no direct coupling to the history of the universe, the value of the density ρ_u depends on the age of the universe. That is, ρ_u depends on how long and at what rate the expansion has been going on. Since, according to Eq. (7.17), G is directly proportional to H, $\rho_u = 3H^2/4\pi G$ is directly proportional to H, too. Consequently the quantities ρ_u, G, H, and $1/R$ are proportional to each other and to $1/t$. And, since ρ_u has to be divided by the factor 40, also G, H, and $1/R$ have to be divided by the same factor. In other words, the universe's age $t = 1/3H$ should be replaced by

$$t = f_t/3H, \tag{0.2}$$

where the numerical constant f_t is of the order of magnitude

$$f_t \approx 40. \tag{0.3}$$

Similarly, Eqs. (5.15), (7.22), and (7.24) have to be replaced by

$$\rho_u = \frac{3H^2}{4\pi f_t G}, \tag{0.4}$$

$$H = f_t/3t, \tag{0.5}$$

and

$$\dot{\rho}_u/\rho_u = \dot{G}/G = \dot{H}/H = -\dot{R}/R = -1/t = -3H/f_t, \tag{0.6}$$

respectively, while Eqs. (7.17), (7.18), and (7.23) do not change.

Now it is easy to understand how the universe's large structures have formed. Since the universe is very old, with its age $t = f_t/3H$ around 200 Gyr, the structures have had plenty of time to evolve. And, since the gravitational force in the young universe was very much stronger than today, microscopic lumps of matter held together by gravity formed rapidly. As the universe grew, the lumps within a sphere of radius

$$r_{\text{tp}} = R/\sqrt{3} = 0.577\,R. \tag{5.10}$$

stuck together in "minigalaxies" while more distant lumps were pushed away so that voids formed between the minigalaxies. As the expansion continued, the structures bound together by the attractive force of gravity grew ever larger at the same time as the — over long distances repulsive — force of gravity created ever vaster voids between the structures.

If today the radius of the universe is $R = 3ct = c \times 600$ Gyr $= 600$ gigalightyear (Glyr), then $r_{tp} = R/\sqrt{3} = 350$ Glyr. However, the formation of large structures and voids ceased a long time ago after the gravitational force had lost most of its earlier strength. Instead, ever smaller, once tightly bound structures have begun to participate in the general expansion so that today gravity only controls galaxies and comparatively dense clusters of galaxies.

It turns out that the lightest Higgs particle (H_e) has a very long half-life, which means that it is practically stable. Consequently, H_e particles produced in stars form invisible dark matter, which flows out from the stars and affects the motion of stars and galaxies [1]. As a result, the factor f_t in Eq. (0.3) above may need to be adjusted downward.

What the model outlined in Sections 5 and 7 does not explain is the age of the universe. Why should the universe be perhaps even 40 times older than the simple model predicts?

(4) Primordial black holes (pages 26, 27, 54, 61)

The decoupling of the photon (or microwave) radiation from matter takes place at a temperature of $T_d \approx 3000$ K, which corresponds to the energy $E_\gamma = kT_d = 0.26$ eV (where $k = 8.617 \times 10^{-5}$ eV/K is the Boltzmann constant).

The simulation (discussed in Appendix F, page 44) of the universe's first phases shows that the average energy of the background photons is $2m_e/10.535 = 97\ 010$ eV (see line following Eq. (E.16) on page 41) when the first proton–electron pair appears, the universe exits its indeterminate quantum phase, and the gravitational force begins to act on particles. It means that the background radiation has lost almost all of its original energy when it decouples from matter (only 26 ppm remains: 0.26 eV/$97\ 010$ eV $= 0.000\ 0026$). Since, as the simulation shows, it is the transfer of energy from radiation to matter that causes c and τ to grow in the global picture, the conclusion is that, after the decoupling, time has ticked at very nearly the same rate in the local and global pictures.

After the decoupling took place, the energy of the background photons has continued to decrease, and the cosmic background radiation (CBR) temperature is today 2.725 K, or about one thousandth of what it was at the time of decoupling. Consequently, the expansion has led to a thousandfold increase in cosmological distances (r) during the time light has traveled freely in the universe.

Since cosmic volumes grow at a constant rate in the global picture (see first paragraph in Appendix F), $t \propto V \propto r^3$, which means that the universe today should be 10^9 times older than it was at the time of decoupling.

Similarly, $G \propto 1/t$, which follows from Eq. (7.17) and Eq. (0.5) above, implies that the gravity of today should be 10^9 times weaker than it was at the time of decoupling. This conclusion is contradicted by observations indicating that the gravitational force should have decreased by far less than fifty percent during the time light has traveled freely. To understand this "age paradox," one must return to the instant when the proton is born and gravity becomes active.

The simulation of the early evolution of the universe suggests that the initial value, G_0, of the gravitational constant was about 4.5×10^{31} times larger than its present-day value.

> When the first proton appears, the universe is about 4×10^{-15} s old (see page 2 above). Assuming that the expansion continues in the same manner as before the appearance of the proton, calculations lead to Eq. (7.22) and the value $t = 1/3H = 5.7$ Gyr for the age of the universe. Division of 5.7 Gyr by 4×10^{-15} s gives the ratio $5.7 \times 10^9 \times 31\ 557\ 926$ s$/4 \times 10^{-15}$ s $= 4.5 \times 10^{31}$.

The smallest mass of a black hole coincides with the so-called Planck mass,

$$M = \sqrt{\hbar c/G}, \tag{0.6}$$

where $\hbar = h/2\pi$ is the reduced Planck constant, and h is Planck's quantum of action. Today, $G/\hbar c = 6.708 \times 10^{-39}$ $(\text{GeV}/c^2)^{-2}$, which gives $M = 1.22 \times 10^{19}$ GeV/c^2 (equal to 2.2 micrograms).

According to Eq. (E.16) on page 41, there are $N_\gamma = 2.786 \times 10^9$ photons surrounding the original proton–electron pair. With $E_\gamma = 97\ 010$ eV, one obtains $N_\gamma E_\gamma = 2.70 \times 10^5$ GeV for the total energy of the photons. For the proton and electron, together with their surrounding photons, to be able to form a black hole, the initial force of gravity must have been $(M/N_\gamma E_\gamma = 1.22 \times 10^{19}\ \text{GeV}/2.70 \times 10^5\ \text{GeV})^2 = (4.52 \times 10^{13})^2 = 2 \times 10^{27}$ times stronger than today. Since the force should have been 22 500 times stronger than that (4.5×10^{31} times stronger than today), and since the background radiation consisted of entangled photon pairs at rest, the logical conclusion is that the photons immediately condensed into a primordial black hole (PBH) with the proton–electron pair at its center.

> **Comment 1.** At its birth, the electron is moving fast relative to the proton. Therefore, it seems more plausible that each massive particle (proton and electron) becomes the nucleus of a black hole. In that case, the PBHs carry electric charge and move slowly relative to each other.

The black hole attempts to grow and swallow as many as possible of the photons surrounding it. However, the gravitational force is a result of the expansion of the universe. And the expansion, in turn, is caused by elementary particles producing space in proportion to their energy content. The particles swallowed by the black hole are isolated from the outer world, and cannot contribute any more to the universe's expansion. Consequently, the larger the portion of the background photons in the black hole is, the more the expansion and the gravitational force decrease.

> **Comment 2.** It may look like a paradox that for example a star collapsing into a black hole retains its gravity even though the content of the black hole no longer contributes to the expansion and to the value of G. However, the apparent paradox has a simple explanation.
>
> The gravitational time dilation means that — in the eyes of an outside observer — processes in an object falling toward a black hole appear to run slower and slower until time finally stops on the hole's event horizon. The fact that time does not run on the surface of the black hole means, in turn, that the gravity of the mass and energy accumulated in the hole continues to affect its surroundings forever.

If, at a given point of time, there are N photons inside the black hole and $N_\gamma - N$ photons outside it, gravity will be N/N_γ as strong as it was before the PBH formed. When the hole has reached its maximum size, evaporation from it will prevent gravity from weakening any further. The balance condition is

$$(N_\gamma - N)E_\gamma/c^2 = \sqrt{\hbar c/(N/N_\gamma)G_0}. \tag{0.7}$$

With $G_0 = 4.5 \times 10^{31}\, G$, where G is the present-day gravitational constant, the condition is fulfilled when the ratio N/N_γ has decreased to

$$N/N_\gamma = 0.000\ 045, \tag{0.8}$$

which means that the very first black hole grows until only 45 ppm of the photons remain free outside the event horizon.

Denote $N/N_\gamma = x$ and $E_\gamma/c^2 = M_\gamma$. Write Eq. (0.7) in the form

$$1/x = (1-x)^2 (N_\gamma M_\gamma)^2 G_0/\hbar c, \quad \text{or} \tag{0.9}$$

$1/\mathrm{x} = (1-x)^2 \times (2\ 786\ 000\ 000 \times 97\ 010 \times 10^{-9})^2 \times 4.5 \times 10^{31} \times 6.708 \times 10^{-39}$
$= (1-x)^2 \times 2.205 \times 10^4$,
and solve it iteratively with $1 - x = 1$ initially:
$(1-x)^2 = 1$: $1/x = 22\ 050$, $x = 0.000\ 045$, $(1-x)^2 = 0.9999$;
$(1-x)^2 = 0.9999$: $1/x = 22\ 048$, $x = 0.000\ 045$.

As the universe expands, small black holes merge into bigger ones with the result that, after its initial abrupt retardation, the expansion continues to slow down and gravity steadily weakens. Holes that remain small evaporate as G decreases.

The evolution of the universe means that the black holes — the building blocks of the fractal structures — grow with time. Today, these building blocks form the centers of the galaxies that became visible when light decoupled from matter.

Compared to many other simulations in astrophysics, it should be a fairly simple task to do a computer simulation showing how the large structures of the universe grow and age with time. All advanced calculations should be done in the local picture, while the rate of expansion should be estimated in the global picture. Therefore the connection between the local and global times is needed. Still, in a first approximative calculation, the time difference may be ignored.

Comment 3. The fact that stars older than 15 Gyr have not been observed suggests that it is the freely traveling light that compresses gas clouds into shining stars.

Comment 4. The fact that protons and electrons exist in large numbers suggests that matter trapped in black holes retains its identity. In other words, the net baryon number B (number of protons and neutrons minus number of antiprotons and antineutrons) and the net lepton number L (number of electrons minus number of positrons) are conserved even during matter's captivity in black holes. This means that the singularity, which in traditional black-hole theory lies at the center of the black hole, in reality does not exist.

Comment 5. The fact that time ceases to run on the surface of a black hole should imply that time stands still inside the hole, too. When gravity decreases, the particles on the hole's surface come to life and evaporate. Consequently, early in the history of the universe there should have been a large number of successively bigger mini black holes that have exploded and caused gravitational waves that might still be observable today.

Comment 6. The hot-big-bang theory's mathematically non-specifiable initial swarm of immensely hot particles is in the present model replaced by a lone, temperatureless and mathematically well-defined primeval particle. And the inflation, which rapidly inflates the universe to enormous proportions only to suddenly stop, is replaced by an abrupt retardation of the expansion followed by a period of very slow expansion — the universe's hundred billion year long, dark and boring infancy — before the expansion again accelerates, the photons decouple from matter, and the first generation of stars ignite.

The illusion of an accelerating universe

The Chandrasekhar limit is about 1.4 solar masses. The limit, which is proportional to $G^{-3/2}$, specifies the critical mass at which white dwarfs explode and form "standard candle" type Ia supernovae. Billions of years ago, gravity was stronger and the Chandrasekhar limit lower than today. Therefore, since there was less material fueling an explosion, and less matter the explosion could eject, old type Ia supernovae shine less brightly than younger ones do.

It should be easy to estimate the rate of decrease in G that is required to explain the observed illusory acceleration of the universe. Further, since according to Eq. (0.6), $\dot{H}/H = \dot{G}/G$, it should be possible to obtain a consistent distance scale and a corrected value for the present-day Hubble expansion rate (which, assuming constant G, is found to be around 1/13.5 Gyr) that hopefully matches the theoretically obtained value of $1/H = 17.2$ Gyr.

Secrets revealed by the muon–electron mass ratio

Let V be an arbitrarily large volume that coexpands with the universe. Denote by E the total energy of the particles within the volume. Leaning on two assumptions,

$$\mathrm{d}V/\mathrm{d}t = \text{constant} \quad \text{and} \quad E = \text{constant},$$

the universe's evolution may be simulated. Two measured mass ratios,

$$m_\tau/m_\mu = 16.8183(27) \quad \text{and} \quad m_\mu/m_e = 206.768\ 2823(52),$$

are used to check and calibrate the simulation. With the help of data produced by the simulation, a theoretical value is obtained for the latter ratio:

$$(m_\mu/m_e)^{\text{th}} = 206.768\ 283\ 185(78),$$

which is nearly two orders of magnitude more precise than the experimental value used in the calibration. The muon–electron mass ratio contains detailed information about the history of elementary particles:

$$m_\mu/m_e = 205.759\ 223 + 1.009\ 816 - 0.000\ 208 - 0.000\ 623 + 0.000\ 074 = 206.768\ 283$$

1966

$1/B\alpha = 205.759\ 223$ is the basic ratio, in which α is the fine-structure constant with $1/\alpha = 137.035\ 999\ 084(51)$ and $B = 0.666\ 001\ 731\ 498$ characterizes the simplest possible solution to the pressureless form of the momentum equation.

1999

The small negative correction ($-0.000\ 2076$) to the mass ratio is a result of the Higgs appearing and conveying mass from the virtual charged leptons $\tau^\pm$, $\mu^\pm$, and $e^\pm$ (which appear in the background photon propagators) to the real quarks forming two pion pairs ($\pi^+\pi^-\ \pi^+\pi^-$) that replace the last decaying electron pairs ($e^+e^-\ e^+e^-$).

2008

The mass brought by the Higgs particles is more than the quarks need to build the proton pair. The neutrino restores the excess mass to the leptons.

2007

The contribution $1/(1-2B\alpha) = 1.009\ 816$ to the muon mass derives from photons that materialize when the spinless muon is succeeded by the spinning electron as bearer of the universe's mass.

2008

The second negative correction ($-0.000\ 6228$) to the mass ratio is a result of Higgs particles conveying three times more mass from the leptons to the quarks when the last decaying pion pair ($\pi^+\pi^-$) is replaced by a proton pair ($p\bar{p}$).

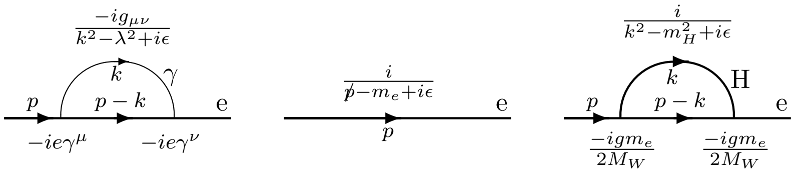
Calculation of mass correction

The expression for the second-order photon and Higgs contributions to the lepton mass,

$$\delta m^{(2)}(\gamma) + \delta m^{(2)}(H) = \ln\frac{\Lambda}{m}\left(\frac{3\alpha}{2\pi} - \frac{3G_F m^2}{8\sqrt{2}\pi^2}\right)m, \tag{0.10}$$

obtained in Appendix C was derived in an indirect way that requires knowledge of the Feynman-parametric formulation of QED [2], which was developed by Toichiro Kinoshita and Predag Cvitanović in 1974. To understand how Eq. (0.10) may be obtained from first principles — that is, from the Feynman rules of SM — one must take a look at these rules.

In the figure, the propagators for the photon, electron, and Higgs are shown above their corresponding particle lines, while the expressions for the photon–electron and Higgs–electron vertices are shown below the electron line:



The notation follows the convention established by James Bjorken and Sidney Drell in the first [3] of their two standard-setting textbooks on quantum field theory (QFT) published in 1964 and 1965, respectively.

Thus, in Feynman's slash notation, $\not p$ is the inner product of the four vector γ and the four momentum p, or

$$\not p = \gamma \cdot p = \gamma^\mu p_\mu = \gamma_\mu p^\mu, \tag{0.11}$$

where the convention of summing over repeated indices is used (e.g., $\gamma^\mu p_\mu = \gamma^0 p_0 + \gamma^1 p_1 + \gamma^2 p_2 + \gamma^3 p_3$). For the time component of a four vector such as p, it holds that $p^0 = p_0$, and for its space components, $p^i = -p_i$ ($i = 1,\ 2,\ 3$). The components p_1, p_2, and p_3 form the momentum vector $\boldsymbol{p}$. The same rules apply to the four vector γ ($\gamma^0 = \gamma_0$ and $\gamma^i = -\gamma_i$ with $(\gamma_1,\ \gamma_2,\ \gamma_3) = \boldsymbol{\gamma}$).

The arrows shown in the figure indicate four momentum — p for the electron, and k for the photon and Higgs. The indices μ and ν indicate that summation over photon and electron polarizations must be performed for the photon–electron loop, while no similar summation is needed for the Higgs–electron loop (the reason for the difference being that the photon is a spin-1 boson and the Higgs a spin-0 boson). For computational reasons, the photon is attributed an infinitesimal mass (λ) that is set equal to zero in final results.

Moving clockwise around the loops and multiplying the expressions with each other, compare with Eq. (8.34) in Ref. [3], one obtains for the integrand associated with the photon–electron loop,

$$I(\gamma) = \frac{-ig_{\mu\nu}}{k^2 - \lambda^2 + i\epsilon}\left(-ie\gamma^\nu\right)\frac{i}{\not p - \not k - m_e + i\epsilon}\left(-ie\gamma^\mu\right), \tag{0.12}$$

and for the integrand associated with the Higgs–electron loop,

$$I(\mathrm{H}) = \frac{i}{k^2 - m_H^2 + i\epsilon} \left(-ig\frac{m_e}{2M_W}\right) \frac{i}{\not p - \not k - m_e + i\epsilon} \left(-ig\frac{m_e}{2M_W}\right). \quad (0.13)$$

The symbol $g_{\mu\nu}$ appearing in the photon propagator is given by the 4×4 matrix [3, p. 281]

$$g_{\mu\nu} = g^{\mu\nu} = \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix},$$

where only the nonzero elements of the matrix are explicitly shown. Similarly, the components of the four vector γ are the Dirac matrices [3, p. 282]

$$\gamma^0 = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}, \ \gamma^1 = \begin{bmatrix} & & & 1 \\ & & 1 & \\ & -1 & & \\ -1 & & & \end{bmatrix}, \ \gamma^2 = \begin{bmatrix} & & & -i \\ & & i & \\ & i & & \\ -i & & & \end{bmatrix}, \ \gamma^3 = \begin{bmatrix} & & 1 & \\ & & & -1 \\ -1 & & & \\ & 1 & & \end{bmatrix}.$$

The fundamental property of the γ matrices is the anticommutation relation

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}, \quad (0.14)$$

from which the rest of their properties may be derived using the fact that $g_{\mu\nu}$ lowers the index of a four-vector component while $g^{\mu\nu}$ raises it;

$$g_{\mu\nu}\gamma^\nu = \gamma_\mu, \quad g^{\mu\nu}\gamma_\nu = \gamma^\mu, \quad g_{\mu\nu}p^\nu = p_\mu, \quad g^{\mu\nu}p_\nu = p^\mu. \quad (0.15)$$

For instance, multiplication of Eq. (0.14) by $p_\mu q_\nu$ yields

$$\not p \not q + \not q \not p = 2p_\mu q^\mu = 2p \cdot q \quad (0.16)$$

(since, being scalar quantities, p_μ and q_ν commute with γ matrices; $\gamma^\nu p^\mu = p^\mu \gamma^\nu$). With $q = p$, the relation simplifies to

$$\not p^2 = p^2. \quad (0.17)$$

Also, readily obtained are the relations

$$\gamma_\mu \gamma^\mu = 4, \quad \gamma_\mu \not p \gamma^\mu = -2\not p, \quad (0.18)$$

the latter via $\gamma_\mu \not p \gamma^\mu = \gamma_\mu \gamma_\alpha p^\alpha \gamma^\mu = (2g_{\mu\alpha} - \gamma_\alpha \gamma_\mu)\gamma^\mu p^\alpha = (2\gamma_\alpha - \gamma_\alpha \gamma_\mu \gamma^\mu)p^\alpha = -2\gamma_\alpha p^\alpha$.

Ignoring the infinitesimal constant ϵ, using $g_{\mu\nu}\gamma^\nu = \gamma_\mu$, and introducing the fine-structure constant α and the Fermi coupling constant G_F via the relations [4, p. 159]

$$e^2 = 4\pi\alpha, \quad G_F/\sqrt{2} = g^2/8M_W^2, \quad (0.19)$$

the integrands may be written

$$I(\gamma) = -4\pi\alpha \frac{\gamma_\mu(\not{p} - \not{k} + m_e)\gamma^\mu}{(k^2 - \lambda^2)((p-k)^2 - m_e^2)} \tag{0.20}$$

and

$$I(\mathrm{H}) = \sqrt{2} G_F m_e^2 \frac{\not{p} - \not{k} + m_e}{(k^2 - m_H^2)((p-k)^2 - m_e^2)} \tag{0.21}$$

when the electron propagator is rewritten according to

$$\frac{1}{\not{p} - m_e} = \frac{1}{\not{p} - m_e} \times \frac{\not{p} + m_e}{\not{p} + m_e} = \frac{\not{p} + m_e}{\not{p}^2 - m_e^2} = \frac{\not{p} + m_e}{p^2 - m_e^2}. \tag{0.22}$$

Before the integrands can be weighed against each other, the numerator in Eq. (0.20) must be simplified. With the aid of Eq. (0.18), the integrand becomes

$$I(\gamma) = 8\pi\alpha \frac{\not{p} - \not{k} - 2m_e}{(k^2 - \lambda^2)((p-k)^2 - m_e^2)}. \tag{0.23}$$

Integration over the four momentum k produces a divergent result for k approaching infinity — hence the UV cutoff mass Λ in Eq. (0.10). The fact that m_H^2 and m_e^2 appear alongside k^2, and m_e alongside k, explains why no particle masses appear in the divergent part of the expression for $\delta m^{(2)}$ (since $m_e/\gamma k$, m_e^2/k^2, and $m_H^2/k^2 \to 0$ for $k \to \infty$).

Division of Eq. (0.21) by Eq. (0.23) shows that in the limit when $k \to \infty$ (and the integral diverges), the ratio between the two integrands is

$$\frac{I(\mathrm{H})}{I(\gamma)} = \frac{G_F m_e^2}{4\sqrt{2}\pi\alpha}, \tag{0.24}$$

which is the same ratio as in Eq. (0.10), but with opposite sign. Consequently, the result obtained in Eq. (0.24) demonstrates a mistake in the original calculation.

References

[1] S. Sundman, *A light Higgs particle explains physics mysteries*, physicsideas.com/Article3v1.pdf (2014).

[2] T. Kinoshita, in *Quantum Electrodynamics*, edited by T. Kinoshita (World Scientific, Singapore, 1990), pp. 218–321.

[3] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[4] W. Greiner and B. Müller, *Gauge Theory of Weak Interactions*, 2nd ed. (Springer, Berlin, 1996).

[5] Fabio Capela, Maxim Pshirkov, and Peter Tinyakov, *Constraints on primordial black holes as dark matter candidates from capture by neutron stars*, Phys. Rev. D **87**, 123524 (2013) [arXiv:1301.4984].